A Smart Assignment Technique with Consideration of Multicriteria Reciprocal Judgments

Zeynab Bahrami Bidoni; Roy George
Department of Computer and Information Systems
Clark Atlanta University
Atlanta, GA
z.bahrami62@gmail.com
rgeorge@cau.edu

Ahmad Makui
Department of Industrial Engineering
Iran University of Science and Technology (IUST)
Tehran, Iran
amakui@iust.ac.ir

***Abstract*— To date the assignment problems are important tasks in recommender systems and one-to-one matching issues through social environments. The various approaches have been proposed to reach these purposes that are normally limited to the considerations of cost or profit incurred by each possible assignment. However most of the time, each of the alternatives at both assignment sides have particular criteria for judging about the other side alternatives, whereby they can evaluate their sufficiency. In this paper, in order to obtain the optimality of both dimensions of assignment we try to consider the concept of efficiency rather than the cost or profit of each possible assignment. Therefore, the efficient assignment is the one that firstly, has the maximum optimality in terms of both dimensions of assignment, and secondly, takes into account the significance of judgment of each assignment from the viewpoint of decision maker. To do this, a compound index would be defined which includes the efficiency related to two-dimensional optimized assignment for the purpose of measuring the performance of each possible assignment. Next, A mathematical programming model for the extended assignment problem is proposed, which is then expressed as a classical integer linear programming model to determine the assignments with the maximum efficiency. A numerical example is used to demonstrate the approach.**

Keywords*— *Assignment Problem; Multicriteria Reciprocal Judgments; Two-dimensional Utility; Total Efficiency in Reciprocal Optimality; Virtual Alternative.

I. INTRODUCTION

The assignment problem is a common term in the theory of linear and network flow. This problem has been proposed in different forms [1] but it is most often considered in form of optimal solution of assigning 'n' jobs to 'n' people in a way that minimum cost or maximum profit would be obtained. You can see some of its usage in [2-6]; in order to find effective and optimal solutions, different algorithm including standard linear programming [7-12], Hungarian algorithm [13], neural network [14], and genetic algorithm [15-19] have been devised. For standard assignment problem, only the cost or the profit of each possible assignment are considered in formulation of the problem; but in real usage, for each possible assignment several types of input resources are usually needed in an assignment problem. Moreover, decision-makers can have several different objectives to achieve for each possible assignment, and the ways to achieve these objectives may conflict with each other. Cambell and Diaby in an article [20] pointed out that demand levels in different departments as well as the number of present workers should be regarded as the input, and the assignment outcomes can affect quality of service and employee satisfaction. They also emphasized that effective utilizing of human resources is of utmost significance in sensitive professions such as nursing.

Bera and Suer also claim that multiple factors can affect the assigning of human resources in the manufacturing cell. Overall, different evaluation units could be used to assess performance measurements of the objectives. These measurements are considered as the output of the problem. The problem can have several incompatible and opposing input and output. In this regard, in an article [22] the author has formulated a problem by considering multiple input and output for each possible assignment, and utilizes data envelopment analysis (DEA) for measuring the efficiency in proposed approach.

Chi-Jen Lin (2011), proposes a labeling algorithm to identify two other sensitivity ranges – Type II and Type III. The algorithm uses the reduced cost matrix, provided in the final results of most solution algorithms for AP, to determine the Type II range which reflects the stability of the current optimal assignment [23]. Birger Raa et al. (2011) In [24] present a MILP model for the integrated BAP–CAP taking into account vessel priorities, preferred berthing locations and handling time considerations. Robert F. Bordley & Stephen M. Pollock (2012) in [25] used an approach that maximizes organizational utility which is assumed to be zero if any of the activities cannot meet its target (or resource allocation). In their approach, utility-based probability maximization (UPM) is a variant of stochastic optimization without recourse.

The standard assignment problem is a particular form of the transportation problem and could be formulated in a linear integer programming of 1-0 [26-27], as follows:

ISBN: 978-1-62561-003-4

$$Min\ (or\ Max) \quad \phi = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}s_{ij}$$

Subject to

$$\sum_{j=1}^{n} s_{ij} = 1\,,\ i = 1,\dots,n$$
$$\sum_{i=1}^{n} s_{ij} = 1\,,\ j = 1,\dots,n \qquad (1)$$
$$s_{ij} = 0\ ,or\ 1$$

In which the decision variable $s_{ij} = 1$ means that 'i' th individual is assigned to 'j' th job, while for $s_{ij} = 0$ no assignment is made. c_{ij} is the cost (or profit) imposed by the assignment. Particular computer software could easily be used to solve above formulated problems as well as to find the set of optimal answers for identifying the minimum cost and maximum profit. But it should be noted that in this formulation, the cost or profit is only regarded for measuring the function and as we mentioned earlier, other criteria rather than profit or cost could be used for measuring the function of assignments.

The basic idea of performing this research has been derived from the assignment problem which encounters in real positions and is not solvable with current methods. The problem is that we want to optimally assign some of employees to some jobs in a way that each of occupations needs some kind of capability and eligibility as evaluation criterion. Meanwhile, manager as decision maker in order to enhance sense of job satisfaction wants to take into account tastes and utility of employees in case of each job. Meanwhile, imposing each person's taste and also qualifications and capabilities needed for every job have different level of importance. Therefore, we deal with assignment problem of two goals: first, to maximize degree of utility in view of each person's taste and second, to maximize degree of utility from the dimension of qualification and competency needed for each occupation according to the priority of each items. As another example we can consider a coach as a decision maker who intends to divide his/her students into different teams in different sports with limited space; in this decision making process he should take into account the qualifications and capabilities required for each sport area as well as the taste of the individuals so that the teams could have the required conditions for success.

Therefore, in this study, the maximum of the total efficiency in obtaining the optimality of both dimensions of assignments would be considered as the criterion of optimal assignment according which this study is organized and you could see what will come next in this paper. Part two would discuss about the overall structure of the model and would provide a definition of the problem. Part three put forward an approach for solving the problem and finding the optimal answer. Part four presents an example to better explain the approach, and finally part five deals with the conclusion of the study.

II. The overall structure of problem

Among the basic concepts required for elaborating the model of the problem, are the three concepts of 'alternative role', 'arbiter role' and 'decision maker role'. When an element has the role of an arbiter, it means that it has some criteria for measurement and can assess and order the opposite alternatives. The element that is being judged has the role of an alternative. The element which directly utilizes the assessments and judgments to the final solution has the role of a decision maker. Therefore, the element that has the role of a decision maker has definitely the role of an arbiter, but the element with the role of an arbiter does not necessarily have the role of a decision maker.

Here, we consider the decision making system as consisting of three distinct types of elements (a component of the decision making system called "element" that could accept one or more role of the tree role of "alternative", "arbiter" or "decision maker"). Both the elements of X, Y have the roles of 'alternative' and 'arbiter' reciprocally, and the third element, that is Decision Maker (DM), has the role of a decision maker which is the one responsible for doing the assignment task (See Fig .1). We assume to have 'k' elements of the 'x' type, each of them are shown as X_i, $i = 1,2,\dots,k$; on the other side we have 'l' elements of the 'y' type that each of them are shown as Y_j, $j = 1,2,\dots,l$ $(k \leq l)$. $C^X = \{c_1^X, c_2^X, \dots, c_r^X\}$ is the set of the references of the attributes related to the assessment of Xs and $C^Y = \{c_1^Y, c_2^Y, \dots, c_s^Y\}$ is the set of the references of attributes related to the assessment of Ys. In this problem each element X_i, $i = 1,2,\dots,k$ takes into account some attributes of C^Y as the criterion of assessment and judgment about all Y_js, and also each of Y_j, $j = 1,2,\dots,l$ has considered a subset of C^X attributes for the sake of measurement and judgment about all X_is. Now DM is the one that makes decision about the assignment of elements of the Y type to the elements of the X type and intends to perform the assignment in a way that the maximum optimality is obtained observing the criteria of the elements of the both sides. It should be noted that each element Y_j could only be assigned to one element X_i and the assignment capacity for each X_i, $i = 1,2,\dots,k$ equals the number of P_i (P_i is Natural number and $\sum_{i=1}^{k} P_i \leq l$).

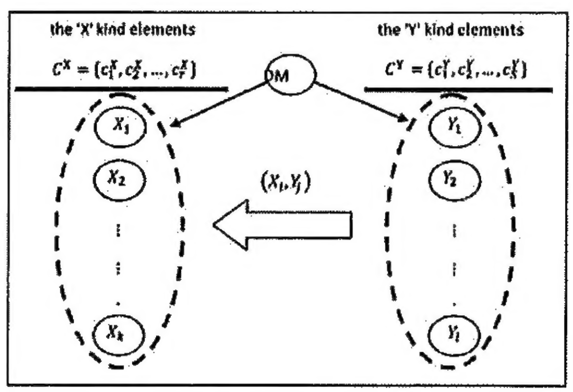

Fig. 1. The structure of assignment model based on multicriteria reciprocal judgments

In addition, DM may attach different significance to judgment of the elements X and Y, therefore, W_X is the significance weight of the elements of the X type and W_Y is the significance weight of elements of the Y type and accordingly $W_X + W_Y = 1$. Also among the elements of X type, the DM may attach different importance to X_is in which case w_{x_i}, $i = 1,2,\dots,k$ is the significance weight of element

 ISBN: 978-1-62561-003-4

X_i in terms of DM in a way that $\sum_{i=1}^{k} w_{x_i} = 1$. In the same vein, about elements of Y type w_{Y_j} , $j = 1,2,\dots,l$ is the significance weight of element Y_j in terms of DM so that $\sum_{i=1}^{l} w_{Y_j} = 1$.

Because of the reciprocity of alternative role in this decision making model, the set of alternatives to be considered in this problem is a two dimensional set which can be viewed as a set of virtual alternatives that are ordered pairs and each of their component is related to each side of the assignment. Therefore the set of $X \times Y$ would be the set of the alternatives to be considered and is represented as follows:

$$A \stackrel{\text{def}}{=} X \times Y = \{(X_i, Y_j) | X_i \epsilon X, Y_j \epsilon Y\} \tag{2}$$

Each alternatives of (X_i, Y_j) from the set of A ($i = 1,2,\dots,k$, $= 1,2,\dots,l$) is interpreted as the assignment of element Y_j to the element X_i . With such a definition, we are dealing with a problem of multi-criteria decision analysis which has $l \times k$ alternatives and one decision maker (DM). It should be noted that each of the X_i, $i = 1,2,\dots,k$ and each of the Y_j , $j = 1,2,\dots,l$ could be arbiter only about alternatives in which one of their components is included. In order to simplify the issue, we define some restrictions of set of A as follows:

$$\forall_{i=1,2,\dots,k}\ A_{x_i} := \{(X_i, Y_j) | Y_j \epsilon Y\} \quad , \\ \forall_{j=1,2,\dots,l}\ A_{y_j} := \{(X_i, Y_j) | X_i \epsilon X\} \tag{3}$$

So with this definition we can say that each element X_i, $i = 1,2,\dots,k$ has the role of arbiter only toward the virtual alternatives of set A_{X_i}, as well as each of the element Y_j , $j = 1,2,\dots,l$ only toward the virtual alternatives of set A_{Y_j}; but DM is the element that has the role of arbiter and decision maker toward all elements of set A. Now, we try to find an algorithm to solve the problem whereby we could obtain the best assignment with maximum optimality in terms of elements of X and Y type.

III. Problem solving approach

Here we are dealing with an assignment problem in which decision maker intends to process the assignment in a way that the maximum optimality could be obtained in terms of both sides of the assignment. Regarding this, first the procedure of ranking which is frequently used in this algorithm would be defined and notated.

A. The ranking procedure:

The purpose of utilizing the ranking procedure is to recognize the criteria, value functions and the mental ideal point of the decision maker on the criteria and to rank the alternatives by measuring the preferable distance of each alternative from the ideal point, so that in terms of the preference amount, the closest alternative to the ideal point would gain the first rank, and in the same way, the remaining alternatives would obtain the next ranks. The symbol of this procedure is written as rank$_*(*,*)$. As an example we could assume that the element b is the arbiter and the set $A = \{a_1, a_2, \dots, a_g\}$ is the set of to-be-considered alternatives. The set $U = \{u_1, u_2, \dots, u_q\}$ is also the reference set of the criteria. Therefore, rank$_b(A, U)$ is the ranking of the set of alternatives A by the arbiter b which is based on the arbitrary criterion of the arbiter among the criteria of the reference set U which is done through these procedures:

Step1. Choosing the criteria: the arbiter would be asked to choose a subset of arbitrary criteria based on which he wishes to do the ranking from the reference set U; the set of chosen criteria is called C.

$$C = \{c_1, c_2, \dots, c_n\} \subseteq U \ ;\ |C| = n \tag{4}$$

Step2. Giving weight to the chosen criteria: in this step we can directly ask the arbiter to provide us with the weight of the criteria and if not possible we can calculate the weights of the criteria through one of the common ways of weight-giving to match in the following conditions.

$$\sum_{i=1}^{n} w_i = 1 \ ,\ w_i > 0 \quad ; \quad W = (w_1, w_2, \dots, w_n) \tag{5}$$

Step3. Identifying the value function related to each criterion by the arbiter: in this phase the arbiter would be asked to identify the mental value function in respect to each criterion. In these functions, the horizontal axe represents the value of outcomes in intended criterion, and the vertical axe is related to the value that those outcomes have for the arbiter. Here, we define 3 aspiration levels for the value size and we ask the arbiter to identify the value size related to outcomes of each criterion based on these levels. These levels are as follows: 1. "quite dissatisfaction" which has the zero value. 2. "quite satisfaction" which has the value of one. 3. "quite surprised " which has the value of two.

If the outcome of a criterion is quite satisfactory for the arbiter, we give value 1 to that outcome in the vertical axe, and in the same vein, for each outcome based on the relative satisfaction it creates for the arbiter, we assign values equal, smaller or larger than 1. The smaller the value is than 1, the more arbiter would be dissatisfaction; and the more it is than 1, the arbiter would be more Surprised. In fact the range of the value function would be between zero to two in which 1 indicates the quite satisfaction and 1 to 2 represents that arbiter is Surprised. As an example, the value function could be as follows:

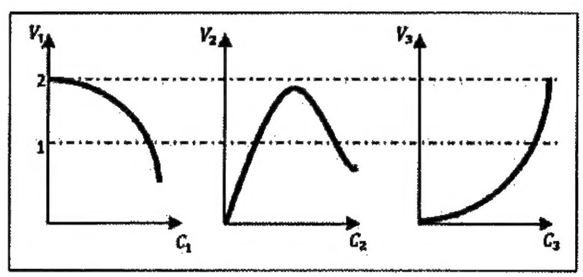

Fig. 2. Three instances of identified value function by the arbiter.

The value function of v_i is a converter that transforms the outcome value obtained from the alternatives a_j, $j = 1,2,\dots,g$ on the c_i criterion to the value defined by the arbiter.

$$v_i^j = F_i\left(C_i(a_j)\right) = v_i(a_j) \ ;\ i = 1,2,\dots,n \ ;\ j = 1,2,\dots,g \tag{6}$$

 ISBN: 978-1-62561-003-4

We define the vector 'V' that has 'n' components as the vector of value functions and in each of its component we put the value function related to one of these criteria.

$$V^j = (v_1^j, v_2^j, \dots, v_n^j) = \left(v_1(a_j), v_2(a_j), \dots, v_n(a_j)\right); \quad j = 1,2,\dots,g \tag{7}$$

Note: in the aggregation model, in order to obtain the whole preferences of the arbiter or decision maker on the alternatives, the assessment criterion for each alternative is considered as a function of value functions and the vector of criteria weight and based on the values obtained from this aggregation function would be ranked in descending order that is the alternative that gain the highest value in the aggregation function would get the rank 1 and the others would be ranked based on the same vein. But get the aggregation function is very difficult because autonomy and dependency status should be among the criteria considered. Sometimes, considering all these relations will not be practical. But this model is proposed a method that to obtain ranking and of aggregation function is not used.

Step4. Formation of n-dimension space with value functions and identification of each alternative a_j ($j = 1,2,\dots,g$) as a point with the coordinates of V^j: we assume to show each of the alternatives of $A = \{a_1, a_2, \dots, a_g\}$ with n-component vector so that the i' th component related to a_j ($i = 1,2,\dots,n$; $j = 1,2,\dots,g$) is the outcome of alternative a_j in the i' th criterion. In this way we could consider the alternatives as points in n-dimension space of criteria. Therefore, each alternative could be displayed with his value functions vector that is alternatives could be considered as points in the n-dimension space of value functions. As an exemplary assumption take n=3 that mean we have 3 criteria so the 3 dimension of criteria and the 3-dimension space of value functions is as follows:

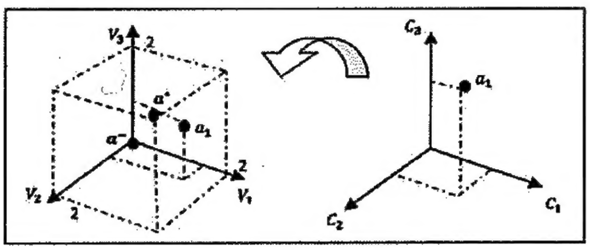

Fig. 3. Definition of the alternatives as the points in the space of criteria and its transference to the value functions space.

Therefore, we would consequently have n-dimension space that each of its dimensions is the identifier of value function related to one of the n-criterion of the arbiter; and each alternative a_j ($j = 1,2,\dots,g$) in this space has the n-component coordinates that could be considered as a spot (point) of this space in a way that the i' th component of each coordinate of alternative equals the value, the outcome of which is obtained in terms of the i' th criterion of arbiter 'b'. The point to be noted is that the value functions space and the coordinates of an alternative in this space is strictly dependent on the idealizations of the arbiter 'b', since the criteria are selected by the arbiter as well as identification of value functions. Therefore, a particular alternative may have quite different coordinates in the space of arbiters' value functions either in terms of the number of the components or in terms of the value of each of the coordinate's components. Consequently we can explain the n-dimension value functions space of arbiter 'b' as the "n-dimensional space of 'b' idealization".

Since the range of the value functions of v_i, $i = 1,2,\dots,n$ is between zero and 2, the ideal point could be considered as a point of the value functions space in which all the components of coordinate is 2, that is the alternative that has obtained the highest value in view of the arbiter and would be represented as $a^* = (2,2,\dots,2)$. In the similar vein the negative ideal point is the one that has obtained the lowest value in view of the arbiter, so all the components of the coordinate equals zero and is displayed as $a^- = (0,0,\dots,0)$. it should be noted that, here we assume a^- has never been a member of the alternatives to be considered, for the simple reason that the occurrence of such a phenomenon i.e., existence of such an alternative that in view of all criteria has the absolute zero value is quite rare and almost impossible. If by chance such an alternative exists, it could be removed from the set of to-be-considered alternatives from the very outset.

Step5. Calculating the closeness of relational preference of alternatives to the ideal point and their ranking based on this index: in this step, we obtain the Euclid distance, between the identifier point of each alternative in value functions space, from the two ideal point and negative ideal point as follows:

$$S_j^* = \sqrt{\sum_{i=1}^{n} w_i * (2 - v_i^j)^2} \quad ; \quad j = 1,2,\dots,g \tag{8}$$

$$S_j^- = \sqrt{\sum_{i=1}^{n} w_i * (v_i^j)^2}; \quad j = 1,2,\dots,g \tag{9}$$

In which for S_j^*, $j = 1,2,\dots,g$ consists of the Euclid distance of alternative a_j from the ideal point and S_j^-, the Euclid distance of alternative a_j from the ideal negative point in the value functions space. Now, in order to rank the set of alternatives A, we define an index termed as "closeness of relational preference to the ideal point" as what you could see below:

$$RPC_b^{a_j} = \frac{S_j^-}{S_j^* + S_j^-} \quad ; \; j = 1,2,\dots,g \tag{10}$$

The $RPC_b^{a_j}$ is the indicator of the closeness of relational preference of alternative a_j to the ideal point of a^* in idealization space of the arbiter b. if $a_j = a^*$, $RPC_b^{a_j}$ equals 1 and when $a_j = a^-$ it equals zero, but since we assume that a^- is not a member of to-be-considered alternatives of A, always we have $0 < RPC_b^{a_j} \le 1$ ($j = 1,2,\dots,g$). the higher is the index for one alternative, the closer the alternative is to the ideal in terms of the preferences of arbiter element 'b', and at the same time it is farther from the negative deal. Finally we order and rank the alternatives of set A, in descending order, from the highest proximity of relational preference to the ideal point, to its lowest proximity.

ISBN: 978-1-62561-003-4

Now for solving this problem and obtaining the most appropriate assignment, we suggest the following phases:

Phase 1: we utilize the ranking procedure for every single elements of X and Y type in arbiter position:

$$\forall_{X_i, i=1,\ldots,k} \quad \mathrm{rank}_{X_i}(Y, C^Y) \quad , \qquad \forall_{Y_j, j=1,\ldots,l} \quad \mathrm{rank}_{Y_j}(X, C^X) \tag{11}$$

Phase 2: in this phase we form the decision matrix of problem by applying the result obtained from the first phase as follows. As we know, we are dealing with a reciprocal judgment and each assignment of a Y type element to a X type element form a to-be-considered alternative which are shown as (X_i, Y_j), $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$. In addition, we want to assess each of these alternatives as tow attributes the first attribute (U^X) is the amount of relative utility of this assignment in terms of X type element, and the second attribute (U^Y) is the amount of the relative utility of this assignment in terms of Y type element. We show the set of these two attributes as $\mathrm{U}^{XY} = \{\mathrm{U}^X, \mathrm{U}^Y\}$. Therefore, the decision matrix structure would be defined as follows:

U^{XY}	U^X	U^Y
(X_1, Y_1)	U^X_{11}	U^Y_{11}
$\vdots$	$\vdots$	$\vdots$
(X_i, Y_j)	U^X_{ij}	U^Y_{ij}
$\vdots$	$\vdots$	$\vdots$
(X_k, Y_l)	U^X_{kl}	U^Y_{kl}

Fig. 4. The structure of decision matrix in assignment model based on the multi-criteria reciprocal judgments.

In which the U^X_{ij} ($i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$) is the result or the outcome of the judgment of X type element about the assignment (X_i, Y_j) and equals the amount of relative utility of element Y_j in terms of element X_i, also U^Y_{ij}, ($i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$) is the result or outcome of the judgment of Y type element about the assignment (X_i, Y_j) and equals the amount of relative utility of element X_i in view of element Y_j. Now the question is that how are these outcomes obtained? For each $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$ we define the outcomes of decision matrix in following way:

$$\mathrm{U}^X_{ij} := w_{X_i} \cdot RPC^{Y_j}_{X_i} \quad ; \quad \mathrm{U}^Y_{ij} := w_{Y_j} \cdot RPC^{X_i}_{Y_j} \tag{12}$$

So for each $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$ We have $0 < \mathrm{U}^X_{ij}\,,\ \mathrm{U}^Y_{ij} \leq 1$.

Phase 3: Now we define an index that could be used as the decision criteria in solving problem. This index is called "total efficiency in reciprocal optimality" and for each alternative (X_i, Y_j), $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$,we show it with E_{ij}. We could consider this index as a linear combination of $\mathrm{U}^X_{ij}, \mathrm{U}^Y_{ij}$, that is if the decision maker (DM) give the weight W_X to the X type element judgment and give W_Y to the judgment of Y type element so that $W_X + \mathrm{W}_Y = 1$, $0 < \mathrm{W}_X, W_Y < 1$, then the linear combination of $E_{ij} := W_X \cdot \mathrm{U}^X_{ij} + W_Y \cdot \mathrm{U}^Y_{ij}$ could be considered as an index for measuring the "total efficiency in reciprocal optimality". However, the point to be noted is that in this definition, the E_{ij} derivative in relation to U^X_{ij} or U^Y_{ij} is a constant value, which is the ratio of the total efficiency changes to the relative utility changes of the alternative is a constant value. While, commonly the closer the amount of relative utility is to 1, and the alternative has higher level of satisfaction, the less the sensitivity would be toward the optimality changes. For this reason, we should define index E_{ij} in a way that it owns this characteristic. Here we define the index E_{ij} for the alternative (X_i, Y_j), $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,l$ in following way:

$$E_{ij} := \frac{1}{1 + \log_{W_Y} \mathrm{U}^X_{ij}} + \frac{1}{1 + \log_{W_X} \mathrm{U}^Y_{ij}} \tag{13}$$

In which the first sentence shows the efficiency of (X_i, Y_j) in X dimension and the second sentence shows the efficiency in Y dimension. Also it is clear that $0 < E_{ij} \leq 2$.

The reasons that confirm the appropriateness of the above definition for E_{ij}:

1. Function $\frac{1}{1+\log_b X}$ (0<b<1) is shown in picture5. Simply we could see that the derivative of this function is positive (ascendant) and its second derivative is negative. In fact the more we get closer from x=1 to x=0 the slope of the graph slowly become lower which is compatible with what is in the mind of the decision maker. Because the more the relative utility is and closer to 1, the less the sensitivity of the decision maker is toward the optimality changes, in other words when the relative utility of an alternative gets higher the speed of the efficiency changes becomes lower.
2. This definition maintains the order and density of the preferences properly, and calculates the efficiency size with regard to the significance weight related to the judgment of each dimension based on the relational preferences.
   - Without disturbing the totality of the problem, we consider the statement related to the efficiency from the dimension of X ($\frac{1}{1+\log_{W_Y} \mathrm{U}^X_{ij}}$), if we consider the sentence related to the efficiency from the Y dimension the change procedure would be the same. Therefore, the consideration of one of these two is sufficient.
   - We assume U^X_{ij} to be constant and increase the W_Y; consequently, as shown in Fig.5, the value of $\frac{1}{1+\log_{W_Y} \mathrm{U}^X_{ij}}$ would decrease; in fact the efficiency would decrease from the X dimension. In other word, if we assume the amount of relative utility of assumed alternative (assignment of Y_j to X_i) in view of X_i judgment as constant and increase the amount of W_Y, in fact we have decreased the significance of judgment in X dimension, because $W_X = 1 - \mathrm{W}_Y$ and it is normal that the efficiency get decreased in X dimension. Also if we decrease W_Y, consequently, the W_X get increased and the value of $\frac{1}{1+\log_{W_Y} \mathrm{U}^X_{ij}}$ would increase and in fact the efficiency would increase from x dimension.

 ISBN: 978-1-62561-003-4

- Now, we keep W_Y as a constant and distinct value and increase the amount of U_{ij}^X. As in Fig. 5, we will see that $\frac{1}{1+\log_{W_Y} U_{ij}^X}$ increases in parallel with increase in U_{ij}^X. That is if we assume W_Y as constant value (in fact, the significance weight related to both dimension is assumed to be constant and specified), based on the graph in Fig. 5, the more the value of relative utility of alternative is in view of X_i judgment, the more the efficiency value would get from the X dimension. The reverse also turns out to be true, that is the decrease of U_{ij}^X for one alternative leads to decrease its efficiency in view of X dimension. However it should be noted that the slope of efficiency changes would decrease with an increase in relative utility value and it is exactly what happens in the decision maker mind, because with increase in the relative utility level, the sensitivity of the decision maker about these changes would decrease and this feature is well considered in the definition of efficiency.
- If U_{ij}^X and U_{ij}^Y have the same value and $W_X > W_Y$, then based on the assumed definition, the efficiency of the alternative from X dimension would get higher than the efficiency of the alternative from the Y dimension.
- If $W_X = W_Y$, meaning that the significance of judgment of X and Y dimension is the same in decision making and $U_{ij}^X > U_{ij}^Y$, then the based on the definition, the efficiency from X dimension would be higher than the efficiency from the Y dimension.

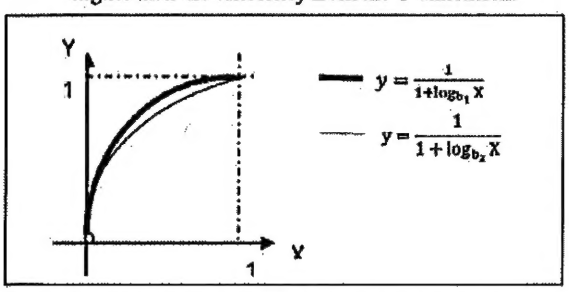

Fig. 5. *The graph of* $Y=\frac{1}{1+\log_b X}$ *when* $0 < b_1 < b_2 < 1$

It should be noted that, here, we aren't looking for the numerical value of efficiency index, but what is important is that this index could properly identify the total preference order based on the two dimension optimality of the to-be-considered alternatives, and since the behavior of defined formulation for E_{ij} is all the time in consistent with reality of decision maker mentality, it seems that this definition is more appropriate and efficient than he basic definition (linear compound). So in this phase for each assignment(X_i, Y_j), $i = 1,2,\dots,k$ and $= 1,2,\dots,l$, the numerical value of E_{ij} would be calculated based on the defined formulation.

Phase 4: In target function of assignment problem, in order to identify the maximum of 2-dimension optimality measurement, the values of E_{ij} would be utilized and the problem would be formulated in following way:

$$MAX \quad \phi = \prod_{i=1}^{k}\prod_{j=1}^{l} E_{ij}^{s_{ij}}$$

Subject to :

$$\sum_{j=1}^{l} s_{ij} = P_i \quad , i = 1,2,\dots,k \qquad (14)$$

$$\sum_{i=1}^{k} s_{ij} \le 1 \quad , j = 1,2,\dots,l$$

$$s_{ij} = 0 \text{ or } 1$$

In which the $s_{ij} = 1$, that is the assignment of element Y_j to the element X_i is done through DM while for $s_{ij} = 0$ no assignment has taken place. Since $0 < E_{ij} \le 2$ and $s_{ij} = 0$ or 1, we could conclude that $0 < \phi$. Therefore, by calculating the logarithme from the target function of ϕ, we could convert the above non-linear problem to the following linear programming problem.

$$MAX \quad \psi = \log\phi = \sum_{i=1}^{k}\sum_{j=1}^{l} s_{ij} \cdot \log E_{ij}$$

Subject to :

$$\sum_{j=1}^{l} s_{ij} = P_i \quad , i = 1,2,\dots,k \qquad (15)$$

$$\sum_{i=1}^{k} s_{ij} \le 1 \quad , j = 1,2,\dots,l$$

$$s_{ij} = 0 \text{ or } 1$$

The above linear programming would certainly have one optimal answer, and that answer would indicate the optimal assignment of the set of Y type element to the set of X type element by DM and with considering the utility of both decision dimensions.

IV. Numerical Example

Here we provide a real experiment about the assignment of job positions to individuals in order to demonstrate the applicability of the suggested approach in which a corporation manager, as a decision maker, requires the decision analysis techniques for assigning three employees (Y_1, Y_2, Y_3) to two jobs of store management (X_1) and finance manager (X_2) so that by considering the utility and employees' interest and also the required competencies of each job, decide on the best assignment in a way that the utility of both sides is being satisfied as far as possible. Here each job is being occupied by one person and the weight factor which the manager allocates for assignment system element is as follows:

$$W_X = 0.7 \quad , \quad W_Y = 0.3$$
$$W_{Y_1} = 0.4 \; , \; W_{Y_2} = 0.4 \; , \; W_{Y_3} = 0.2$$
$$W_{X_1} = 0.6 \quad , \quad W_{X_2} = 0.4$$

In order to extract the required data, we have designed some simple question forms, which were customized to estimate alternative's score on each criterion. In these forms, the person had selected a number in range 1 to 5 for each question to demonstrate his/her preference, and total average number was final score, which is converted in range o to 1.

ISBN: 978-1-62561-003-4

TABLE I. INTRODUCTION OF THE SET OF CRITERIA C^X

C^X	C_1^X	C_2^X	C_3^X	C_4^X
criteria	Salary and benefits	Responsibility amount	Nature of work	Popularity in that department

TABLE II. INTRODUCTION OF THE SET OF CRITERIA C^Y

C^Y	C_1^Y	C_2^Y	C_3^Y	C_4^Y
criteria	Management power	Job-related education	Job-related experience	Public Relations

TABLE III. WEIGHT-GIVING TO CRITERIA FOR JUDGMENT ABOUT YS IN VIEW OF XS

	C_1^Y	C_2^Y	C_3^Y	C_4^Y
X_1	0.4	0.2	0.3	0.1
X_2	0.4	0.1	0.4	0.1

TABLE IV. WEIGHT-GIVING TO CRITERIA FOR JUDGMENT ABOUT XS IN VIEW OF YS

	C_1^X	C_2^X	C_3^X	C_4^X
Y_1	0.5	0.3	0.2	-
Y_2	0.4	0.2	0.2	0.1
Y_3	0.3	-	0.2	0.5

TABLE V. THE VALUES OF Y_jS ON CRITERIA IN VIEW OF X_1

	C_1^Y	C_2^Y	C_3^Y	C_4^Y
Y_1	1	0.7	0.8	0.1
Y_2	0.2	0.3	0.2	0.1
Y_3	0.2	0.4	0.3	0.5

TABLE VI. THE VALUES OF Y_jS ON CRITERIA IN VIEW OF X_2

	C_1^Y	C_2^Y	C_3^Y	C_4^Y
Y_1	1	0.2	0.4	0.1
Y_2	0.2	0.5	1	0.1
Y_3	0.2	0.4	0.5	0.5

TABLE VII. THE VALUES OF X_iS ON CRITERIA IN VIEW OF Y_1

	C_1^X	C_2^X	C_3^X
X_1	0.8	0.4	0.7
X_2	1	0.3	0.3

TABLE VIII. THE VALUES OF X_iS ON CRITERIA IN VIEW OF Y_2

	C_1^X	C_2^X	C_3^X	C_4^X
X_1	0.7	0.3	0.2	0.1
X_2	1.2	0.6	1	0.4

TABLE IX. THE VALUES OF X_iS ON CRITERIA IN VIEW OF Y_3

	C_1^X	C_3^X	C_4^X
X_1	0.4	0.2	0.3
X_2	0.7	1	0.7

TABLE X. DECISION MATRIX

U^{XY} / A	U^X	U^Y
(X_1,Y_1)	0.223	0.134
(X_1,Y_2)	0.062	0.096
(X_1,Y_3)	0.121	0.032
(X_2,Y_1)	0.128	0.138
(X_2,Y_2)	0.122	0.188
(X_2,Y_3)	0.078	0.076

TABLE XI. THE TOTAL EFFICIENCY IN RECIPROCAL OPTIMALITY FOR EACH POSSIBLE ASSIGNMENT

Index / A	E_{ij}	$\log E_{ij}$
(X_1,Y_1)	0.596	- 0.225
(X_1,Y_2)	0.434	- 0.362
(X_1,Y_3)	0.457	- 0.34
(X_2,Y_1)	0.522	- 0.282
(X_2,Y_2)	0.54	- 0.268
(X_2,Y_3)	0.442	- 0.354

At this stage, we solve the following linear programming to achieve the assignment with maximum efficiency.

$$MAX \quad \psi = \log \phi = -0.225\, S_{11} - 0.362\, S_{12} - 0.34\, S_{13} - 0.282\, S_{21} - 0.268\, S_{22} - 0.354\, S_{23}$$

Subject to:

$$S_{11} + S_{12} + S_{13} = 1$$
$$S_{21} + S_{22} + S_{23} = 1$$
$$S_{11} + S_{21} \leq 1$$
$$S_{12} + S_{22} \leq 1$$
$$S_{13} + S_{23} \leq 1$$
$$S_{ij} = 0 \text{ or } 1$$

The optimal answer of this linear programming is: $S^* = (1,0,0,0,1,0)$. That is only amount of S_{11} and S_{22} are 1 and This means that (According to the proposed method in this study) assigning individual Y_1 to position of store management and individual Y_2 to position of finance manager were appropriate decision With regard to judgments of two fronts of assignment. In practice, after assigning new managers based on obtain results. The satisfaction survey (by question forms) demonstrates satisfaction in these two departments over %75 increased than before on both upper managers and employees' levels.

CONCLUSIONS

In this study with proposing a novel viewpoint on the basis of existing reciprocal system of judgment between the alternatives of the both side of assignment, the objective would be to maximize the assignment efficiency in obtaining the two dimension optimality with which cost and profit gets substituted which was considered in standard assignment problem, and for this purpose, a compound index was defined for measuring the function of each possible assignment in problem formulation. Then a mathematical programming model was proposed for problem solution and for determining the assignment with maximum efficiency it was transformed to a classic linear programming model.

 ISBN: 978-1-62561-003-4

REFERENCES

[1] Pentico David W. Discrete Optimization Assignment problems: A golden anniversary survey. European Journal of Operational Research 2007; 176:774–793.

[2] Soumis F, Ferland j, Rousseau J. A model for large-scale aircraft routing and scheduling problems. Transport. Res. Part B: Meth 1980;14(1):191–201.

[3] Campell J.F, Langevin A.The snow disposal assignment problem. J. Oper. Res. Soc 1995; 48: 919–929.

[4] Leblanc L.J, Farhangian K. Efficient algorithm for solving elastic demand traffic assignment problem and mode split-assignment problem. Transport. Sci 1981; 15 (4): 306–317.

[5] Dessouky M.M, Kijowski B.A. Production scheduling on single-stage multiproduct batch chemical process with fixed batch sized. IIE Trans 1997; 29 (5): 399–408.

[6] Mckeown P, Workman B. A study in using linear programming to assign students to schools. Interfaces 1976; 6 (4): 101–96.

[7] McGinnis L.F. Implementation and testing of a primal-dual algorithm for the assignment problem. Oper. Res. 1983; 31(2):277-291.

[8] Balinski M.L. A competitive (dual) simplex method for the assignment problem. Math. Program 1986;34 (2) :125–141.

[9] Hung M.S, Rom W.O. Solving the assignment problem by relaxation. Oper. Res. 1980;28 (4): 969–982.

[10] Barr R.S, Glover F, Klingman D. The alternating basis algorithm for assignment problems. Math. Program 1977;13 (1) : 13–1.

[11] Volgenant A. Discrete Optimization Solving the k-cardinality assignment problem by transformation. European Journal of Operational Research 2004;157: 322–331.

[12] Deng Kui Huang, Huan Neng Chiu, Ruey Huei Yeh, Jen Huei Chang, A fuzzy multi-criteria decision making approach for solving a bi-objective personnel assignment problem. Computers & Industrial Engineering ;2008.

[13] Kuth H.W. The Hungarian method for the assignment problem. Nav. Res. Log. 1955; 2: 83–97.

[14] Eberhardt S.P, Duad T, Kerns A, Brown T.X, Thakoor A.P. Competitive neural architecture for hardware solution to the assignment problem. Neural Networks. 1991; 4(4):431–442.

[15] Gunawan Aldy, Ng K.M, Ong H. L. A Genetic Algorithm for the Teacher Assignment Problem for a University in Indonesia. Information and Management Sciences. 2008; 19(1): 16-1.

[16] Anshuman Sahu, Rudrajit Tapadar, Solving the Assignment problem using Genetic Algorithm and Simulated Annealing. IAENG International Journal of Applied Mathematics. 36:1, IJAM_36_1_7.

[17] Avis D, Devroye L. An analysis of a decomposition heuristic for the assignment problem. Oper. Res. Lett. 1985; 3 (6): 279–283.

[18] Linzhong Liu, Xin Gao, Fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm. Applied Mathematical Modelling 2009; 33: 3926–3935.

[19] Ahuja R.K, Orlin J.B, Tiwari A. A greedy genetic algorithm for the quadratic assignment Problem. Computers & Operations Research 2000; 27: 917-934.

[20] Campbell G.M, Diaby M. Development and evaluation of an assignment heuristic for allocating cross-trained workers. Eur. J. Oper. Res. 2002;138 :12–9.

[21] Su¨er G.A, Bera I.S. Optimal operator assignment and cell loading when lot-splitting is allowed. Comput. Ind. Eng. 1998; 35 (3–4) : 431–434.

[22] Chen L-H, Lu H-W. An extended assignment problem considering multiple inputs and outputs. Applied Mathematical Modelling 2007;31: 2239–2248.

[23] Chi-Jen Lin. A labeling algorithm for the sensitivity ranges of the assignment problem. Applied Mathematical Modelling. October 2011; 35(10): 4852–4864.

[24] Birger Raa, Wout Dullaert, Rowan Van Schaeren. An enriched model for the integrated berth allocation and quay crane assignment problem. Expert Systems with Applications. October 2011;38(11):14136–14147.

[25] Robert F. Bordley , Stephen M. Pollock. Assigning resources and targets to an organization's activities. European Journal of Operational Research. 1 August 2012; 220(3): 752–761.

[26] Pfaffenberger R.C, Walker D.A. Mathematical Programming for Economics and Business. first ed., The Iowa State University Press ;1976.

[27] Gass S.I. Linear Programming. fifth ed.,McGraw-Hill Book Company;1984.